Critical properties of the two-dimensional planar spin model in the presence of p-fold random anisotropy

1991 J. Phys. A: Math. Gen. 24 L1299

(http://iopscience.iop.org/0305-4470/24/21/008)

J. Phys. A: Math. Gen. **24** (1991) L1299–L1304. Printed in the UK

LETTER TO THE EDITOR

Critical properties of the two-dimensional planar spin model in the presence of *p*-fold random anisotropy

P Reed

School of Computer Studies and Mathematics, Sunderland Polytechnic, Sunderland SR1 3SD, UK

Received 31 July 1991

Abstract. The two-dimensional planar spin model in the presence of p-fold random anisotropy has been investigated using Monte Carlo simulation and finite size scaling. The system has been examined for $p=2, 3, 4, 5, 6$ and lattice sizes $N=4^2, 8^2, 16^2$. The critical behaviour is found to depend on the value of p and is of two types. For $p=3, 5, 6$ the results indicate a transition to a quasiferromagnetic state at the same temperature as the pure system. At lower temperatures there is evidence for a second transition; however, the system remains quasiferromagnetic. For $p=2, 4$ the behaviour is different and results suggest a second-order transition to a spin glass phase and no quasiferromagnetism.

The critical properties of the planar spin model (also called the XY model) in the presence of p-fold anisotropy has been investigated by Monte Carlo simulation and finite size scaling. The Hamiltonian for this system is

$$H=-J\sum_{\langle ij\rangle}\cos(\phi_i-\phi_j)-D\sum_i\cos(p\phi_i-\theta_i). \tag{1}$$

In the above $\{\phi_i\}$ are site variables taking values between 0 and 2π and $\{\theta_j\}$ are quenched in random anisotropy axes uniformly distributed in the range 0 to 2π. Interaction is between nearest neighbours and the system is confined to a two-dimensional square lattice with periodic boundary conditions.

For the pure system with $D=0$ the critical properties of Hamiltonian (1) are believed to be well understood [1, 2]. In this case the system undergoes a single phase transition from a high-temperature paramagnetic phase to a low-temperature quasiferromagnetic phase. A quasiferromagnetic phase is characterized by an infinite correlation length and algebraic decay of pair correlation function, and also a zero ferromagnetic order parameter. Renormalization group and other calculations indicate a transition temperature of $\pi/2$. However, subsequent simulation [3] has identified the transition temperature as $0.89J$ but otherwise confirms the overall picture of the critical properties.

For non-zero values of D each site variable resides in a random anisotropy field with p equivalent axes. The effects of such fields have been carefully investigated by Cardy and Ostlund [4] within the methodology of replicas and renormalization group. Their results indicate an interesting phase structure. For a range of temperature given by

$$\frac{4\pi}{p^2}<T<\frac{\pi}{2} \tag{2}$$

the randomness is predicted to be irrelevant. This implies the existence of a temperature range within which the system remains quasiferromagnetic as in the pure system. For $T<4\pi/p^2$ the randomness is predicted to destroy quasiferromagnetism; however, the overlap between replicas does not go to zero indicating that the low-temperature phase may be 'glassy'. This picture of the critical properties is seen from (2) to be valid for $p>\sqrt{8}$. For p less than this the intermediate quasiferromagnetic phase vanishes.

The nature of the low-temperature transition out of the quasiferromagnetic phase was also considered by Cardy and Ostlund. Interestingly their calculations predict that at low temperatures the coupling between vortices renormalizes to negative values. This instability at long length scales is interpreted as a signature of a possible first-order transition. In this picture $p=4$ appears as a special case. For $p<4$ the vortex density renormalizes to zero before the instability in vortex coupling is established, while this is not the case for $p>4$. Therefore $p=4$ is a special case above which Cardy and Ostlund speculated that the transition at low temperatures may be first order.

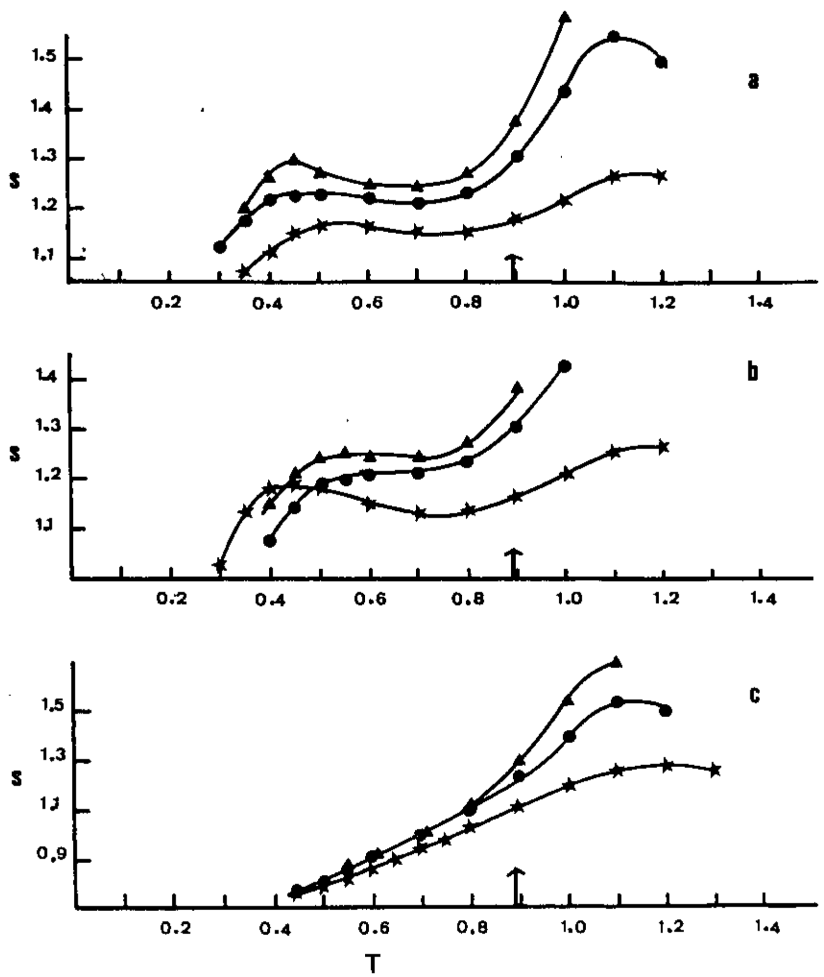

Figure 1. Specific heat S against temperature T: (a) is for $p=6$, (b) is for $p=5$, (c) is for $p=3$. ★ $=4^2$, ● $=8^2$ and ▲ $=16^2$. The Kosterlitz–Thouless transition temperature T_{KT} is indicated by the arrows.

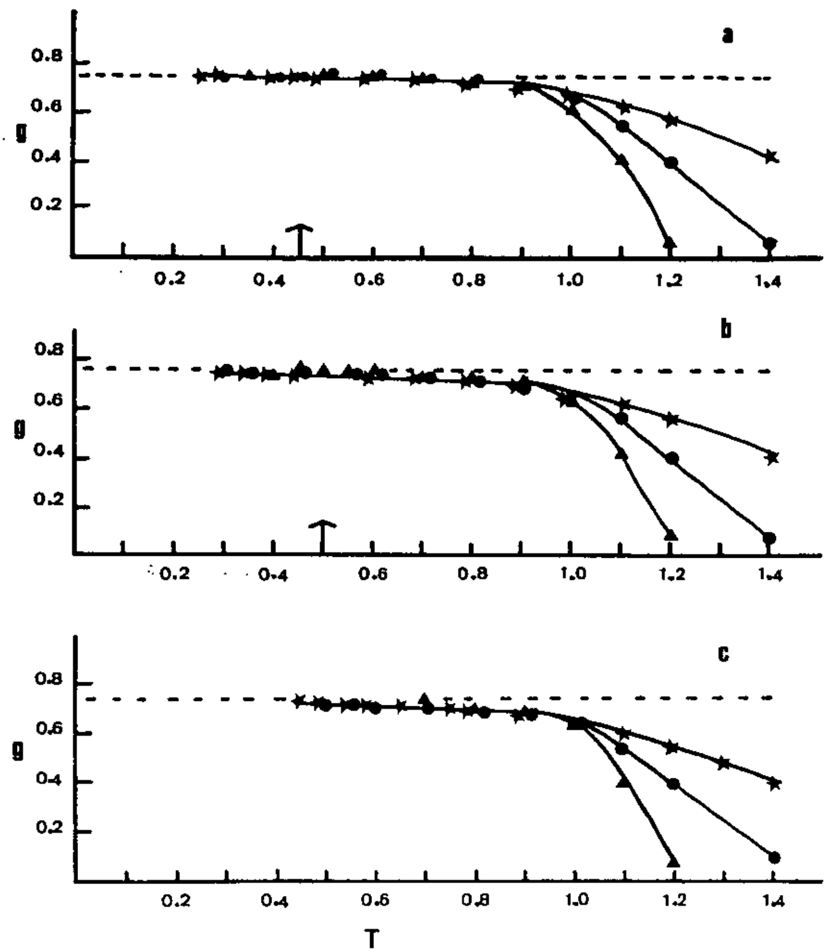

Figure 2. Plots of g against temperature T. The locations of the specific heat peak (from figure 1) are indicated by arrows. The broken line is g calculated for the pure system from [7]: (a) is for $p=6$, (b) is for $p=5$ and (c) is for $p=3$. ● $=4^2$, ● $=8^2$ and ▲ $=16^2$.

Monte Carlo simulation and finite size scaling have been used to probe the system using $J=D=1$ in Hamiltonian (1). The simulation has been done for $p=3$, 4, 5, 6, and also $p=2$ which is outside the range of the predictions of Cardy and Ostlund. The results reported indicate a somewhat different phase structure from that previously expected.

The specific heat has been calculated since it provides an indication of the order of a transition, particularly a first-order transition. The nature of the phases, whether spin glass, quasiferromagnetic or paramagnetic, has been examined by calculating the overlap between non-interacting identical replicas of the system. This is defined to be

$$Q'(t)=\frac{1}{N}\sum_i \cos(\phi_i^1(2t_0+t)-\phi_i^2(2t_0+t)) \tag{3}$$

where ϕ_i^1 and ϕ_i^2 are the values of the site variables in two independent replicas. t_0 is the time used for relaxing the system before calculating $Q'(t)$. N is system size and

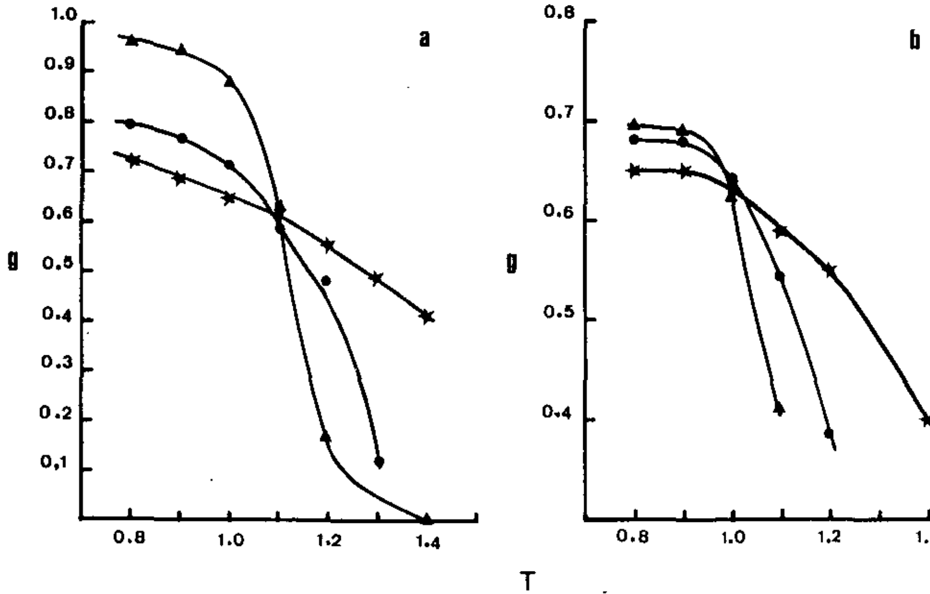

Figure 3. g against temperature T: (a) is for $p=4$ and (b) is for $p=2$. ★ $=4^2$, ● $=8^2$ and ▲ $=16^2$.

for scaling purposes $N=4^2$, 8^2 and 16^2 have been used. Using (3) the distribution of $Q'(t)$ was calculated using

$$p'(q)=\frac{1}{M}\left[\sum_{\tau=1}^{M}\delta\left\{q-Q'\left(\frac{\tau t_0}{M}\right)\right\}\right] \tag{4}$$

where [ ] denotes average of different replicas and M is the number of measurements.

As with most problems involving randomness the problem of metastability and long relaxation times affected this simulation. As a criterion to ensure adequate relaxation times the same criterion as employed by Bhatt and Young [5] in their extensive study of the Edwards–Anderson spin glass was used. To do this the overlap between the same replica at different times is calculated. This is defined as

$$Q(t)=\frac{1}{N}\sum_i \cos(\phi_i^1(t_0)-\phi_i^1(2t_0+t)). \tag{5}$$

The distribution of $Q(t)$ is calculated as

$$P(q)-\frac{1}{M}\left[\sum_{\tau=1}^{M}\delta\left\{q-Q\left(\frac{\tau t_0}{M}\right)\right\}\right]. \tag{6}$$

Equations (4) and (6) give two distributions for q which should be identical in the limit $t_0\rightarrow\infty$. From distributions (4) and (6) the expectation value of q^2 is calculated. Only those results from which these two calculated values of the expectation of q^2 agree are reported here. This imposed quite a severe restriction on the simulation and runs of 4.5 million Monte Carlo steps were used for all temperatures below 0.9.

The results that follow were calculated using $p'(q)$ and the field averaging was done using 256, 64 and 16 replicas for system sizes $N=4^2$, 8^2, 16^2 respectively. From

$P'(q)$ the scaling function g defined by

$$g=\frac{1}{2}\left\{3-\frac{\langle q^4\rangle}{\langle q^2\rangle^2}\right\} \tag{7}$$

was calculated, where $\langle\,\rangle$ denotes average over $P'(q)$. g is a convenient scaling function for detecting both spin glass and quasiferromagnetic order. It has the scaling form

$$g=g((\sqrt{N})^{1/\nu}(T-T_c)). \tag{8}$$

Care must be exercised in interpreting g since in the quasiferromagnetic phase g has an asymptotic value of 0.75 and this behaviour must be distinguished from true spin glass freezing.

The results naturally divide into two types, depending on the value of p. Firstly consider $p=3$, 5, 6. Results for the specific heat are shown in figure 1. The location of the Kosterlitz–Thouless transition temperature T_{KT} is indicated by an arrow. For $p=5$, 6 there is a peak or shoulder in the specific heat at temperatures below T_{KS}. This behaviour is indicative of a second low-temperature transition as predicted by Cardy and Ostlund. A first-order transition is characterized by a growth in peak height proportional to N. No such growth is observable in figures 1(a) or 1(b) and only a weak transition is evident. It may be that the system sizes used here are insufficient to exhibit the asymptotic behaviour which could be first order. However, simulations done for the case $p=6$ only [6] and on system sizes up to $N=128^2$ only indicate the presence of a shoulder in the specific heat at about $T=0.45$.

The results for $p=3$ are shown in figure 1(c). No low-temperature peak or shoulder is evident, confirming the prediction of different behaviour above and below $p=4$ although no indication of a first-order transition has been found.

Plots for g are shown in figure 2. The curves for all system sizes coalesce at T_{KT} and remain concurrent throughout the accessible temperature range. Further, g rapidly saturates at the value of about 0.75 characteristic of a quasiferromagnet. For comparison plots of g calculated for the pure system in the spin wave approximation [7] are also shown. It is seen that these curves are virtually coincident with the Monte Carlo data. The positions of the peaks in the specific heat are indicated by arrows in figure 2. It is seen that there is no change in g at these temperatures that might be the signature of a change to either a paramagnetic or frozen spin glass state. It could be that such a change occurs at lower temperatures and the specific heat shoulder is only a precursor. However, the indications here are that the system remains quasiferromagnetic throughout the temperature range with only a change in short-range order.

For $p=2$, 4 the results are different. The results for g are shown in figure 3. Here it is seen that the curves intersect and then diverge. Thus there is a critical point at about $T=1$ for $p=4$ and $T=1.1$ for $p=2$. The divergence of the curves below these temperatures indicate that the low-temperature phase is not quasiferromagnetic. Figure 3 is typical of the behaviour expected for a transition to a spin glass state and is similar to data obtained for the four-dimensional Edwards–Anderson Ising spin glass [8].

At lower temperatures than those shown in figure 3 there is slight evidence for a change in behaviour of g, perhaps indicating other critical behaviour. However, the numerical data is too weak on this point and this suggestion is put forward only as a speculation.

I thank the SERC for their support and also John Steel and the staff of the Centre for Parallel Computing at London University for much-valued help and assistance.

References

[1] Kosterlitz J M and Thouless D J 1973 *J. Phys. C: Solid State Phys.* **6** 1181
[2] Young A P 1978 *J. Phys. C: Solid State Phys.* **11** L453
[3] Tobochnik J and Chester G V 1979 *Phys. Rev.* B **9** 3761
[4] Cardy J L and Ostlund S 1982 *Phys. Rev.* B **25** 6899
[5] Bhatt R N and Young A P 1985 *Phys. Rev. Lett.* **54** 924
[6] Reed P 1989 *J. Phys.: Condens. Matter* **1** 3037
[7] Berezinskii V L 1970 *Sov. Phys.-JETP* **32** 493
[8] Bhatt R N and Young A P 1987 *Phys. Rev.* B **37** 5606